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FROM SYLLOGISM TO LOGICISM: WAS ARISTOTLE THE FIRST LOGICIST?

Abstract

The question, "Was Aristotle the first logicist?", may appear anachronistic and elicit skepticism since the doctrine of logicism as a fully-fledged idea emerged only in the nineteenth century in the context of the debates surrounding the foundation of mathematics. Indeed, Bertrand Russell credits Gottlob Frege with being the first in "logicising" mathematics (Russell 1919, p. 7), where the thesis espouses that mathematical concepts and propositions are ultimately reducible to or derivable from a number of fundamental logical concepts and principles. However, anachronistic appearances aside, in a fresh reexamination of some of the specific Aristotelian texts in *Metaphysics* and *Prior Analytics*, and especially focusing on Aristotle's particular remarks on the status and significance of the principle of non-contradiction, one may textually argue for a nascent and burgeoning form of logicism in Aristotle, albeit within a much larger metaphysical context than mathematics.

Keywords

Logicism, Metalogic, Principle of Non-Contradiction, Skepticism, Syllogism

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The French historian, Lucien Febvre, in one of his seminal work tells his readers that, from a historiographical point of view, in looking at past figures, the problem is not so much "to catch hold of a man" in isolation from his contemporaries or, just because a certain passage in his work fits in with the direction of one of our own modes of thinking, to decide that he fits under one of the rubrics we use nowadays for classifying those who do or do not think like us. Rather, "the problem is to determine what set of precautions to take and what rules to follow in order to avoid *the worst of all sins, the sin that cannot be forgiven – anachronism*" (emphasis added).[1] Now, judging by the extraordinary expanse of time amounting to more than two millennia between Aristotle and the advent of logicism in the second half of the nineteenth century, as well as the apparent unavailability of the logicist conceptual wherewithal in the ancient world, any question along the lines of the subtitle of this paper, 'Was Aristotle the First Logicist?', is obviously nothing short of the blatant anachronism that Febvre exhorts his readers to avoid. In the realm of ideas, using Febvre's similes for our context, "it is like giving Diogenes an umbrella and Mars a machine gun".[2] Yet my contention here is that there may be mitigating circumstances where the attribution of logicism to Aristotle might not after all be misguided and guilty of anachronism.

To begin the endeavor, before locating the presumed logicist landmarks in the Aristotelian text, the first port of call is to look at the genesis and germination of logicism. Although Russell traces the first explicit and intentional implementation of logicism to the works of Gottlob Frege (1848-1925),[3] recent scholarship on the history of logicism seems to put equal, if not occasionally more, emphasis on the pioneering works of Richard Dedekind (1831-1916) and Giuseppe Peano (1858-1932).[4] However, Russell himself in an earlier work suggests that the trail of the idea of logicism stretches back to Gottfried Wilhelm Leibniz (1646-1716) and remarks that the general doctrine underpinning the idea "was strongly advocated by

[1] Febvre (1982) p. 5.
[2] *Ibid*., p. 353.
[3] Russell (1919).
[4] See, for example, Demopoulos & Clark (2007), Franchella (2019), Reck (2013), Stein (1998).

Leibniz".[5] Yet Max Black seems to take umbrage at Russell's overestimation of the Leibnizian contribution in this context and offers the somewhat conservative characterization that Leibniz's "work contained the germ of" the logistic conception.[6] Yet Frege himself in his classic logicist landmark, *The Foundations of Arithmetic*, approvingly quotes Leibniz that "algebra derives its advantages from a much higher art, namely, true logic".[7] Indeed, in *New Essays on Human Understanding*, Leibniz observes that "geometer's logic – that is, the methods of arguments which Euclid explained and established through his treatment of propositions – can be regarded as an extension or particular application of general logic".[8]

In tracing the logicist threads of the Leibnizian corpus, where historically the first explicit Aristotelian connections appear on the horizons of logicism, Russell highlights Leibniz's idea of *characteristica universalis* or "universal mathematics":

> This was an idea which he cherished throughout his life, and on which he already wrote at the age of 20. He seems to have thought that the symbolic method [...] could produce everywhere the same fruitful results as it has produced in the sciences of number and quantity.[9]

Russell then goes on to say that for Leibniz the "Universal Characteristic seems to have been something very like the syllogism".[10] In fact, Leibniz himself portrays the significance of the Aristotelian syllogism in the following way:

[5] Russell (1996) p. 5.
[6] Black (1958) p. 16.
[7] Frege (2007) p. 31. John Austin renders the quotation from Leibniz in his translation of Frege thus: "the benefits of algebra are due to its borrowings from a far superior science, that of the true logic." See Frege (1978) p. 21e.
[8] Leibniz (1985) p. 370.
[9] Russell (1958) p. 169. Similarly, in one of his unpublished papers dating back to 1880-81, Frege notes that this idea of Leibniz is one of: "a profusion of seeds of ideas [...] that is now to all appearances dead and buried [but] will one day enjoy a resurrection" (Frege 1979, pp. 9-10) and sees his own work in *Begriffschrift* published in 1879 as "a fresh approach to" it in anticipation of the implementation of his logicist agenda".
[10] Russell (1958) p. 170.

> I hold that the invention of the syllogistic form is one of the finest, and indeed one of the most important, to have been made by the human mind. It is a kind of universal mathematics whose importance is too little known. It can be said to include an *art of infallibility* [...].[11]

And the profuse portrayal is expanded to such an extent that Leibniz makes his fictional representative of John Locke in *New Essays on Human Understanding*, Philalethes, backtrack from his dismissal of the syllogism, and to admit that:

> I am beginning to form an entirely different idea of logic from my former one. I took it to be a game for schoolboys, but I now see that, in your conception of it, it involves a sort of universal mathematics.[12]

The significance of the relationship between Aristotle's syllogistic formalization and Leibniz's *ars charateristica universalis* can be better appreciated when it is set against the backdrop of a number of cardinal features of the logicist program. First, for the logicism project to get off the ground, the initial necessary step is to set up a formal deductive system of logic adequate for formalizing the reasoning of one domain into another one. Specifically, in the case of Fregean logicism and its recent descendants in the form of neo-logicism, the formal deductive system must possess the ability to formalize mathematical reasoning. This constitutes the principal *prerequisite* or *precondition* at the implementation of logicism, ad this is, indeed, where Aristotle's syllogistic formalization looms large in the question of his logicist inklings and tendencies.

Secondly, the logicist program involves an unequivocal and unambiguous process of *conceptual reduction* whereby the concepts of the prospective target domain for reduction can be defined in terms of logical concepts. Accordingly, in the preface to *The Principles of Mathematics*, Russell explicitly sets out a twofold task as one of the main objectives of his logicism, whereby

[11] Leibniz (1985) p. 478. In a letter dating to 1696 to Gabriel Wagner on the value of logic against Wagner's anti-scholasticist attack on Aristotelian logic, Leibniz interestingly describes Aristotle in his attempt at syllogistic formalization as the first one to write mathematically outside of mathematics. See Loemker (1969) p. 465: "It is certainly no small matter that Aristotle reduced these forms [paralogisms] to unerring laws, having been the first actually to write mathematically outside of mathematics."

[12] Leibniz (1985) pp. 486-87.

the first fold is to provide "the proof that all mathematics deals exclusively with concepts definable in terms of a very small number of fundamental logical concepts".[13] This clearly captures the second characteristic of the logicist prospectus.

As for the second fold of Russell's first objective in *The Principles of Mathematics*, namely, to demonstrate that all the propositions of pure mathematics "are deducible from a very small number of fundamental logical principles",[14] there arises a third trait of logicism in terms of disambiguating or determining the *scope* of the logicist implementation. Neil Tennant suggests that the claim of logicist reduction can be understood in one of two senses: either in the *strong* sense of claiming that all *truths* of the reduced domain comprise a subset of logical truth or in the *weak* sense of claiming that all *theorems* of the reduced domain comprise a subset of logical truth.[15] Judging by the high hopes and ambitions that Leibniz harbors for his dream of *ars charateristica universalis*, it might not be that controversial to attempt to determine where Leibniz stands on this distinction. But, in the case of Aristotle, should he turn out to be a logicist after all, the verdict might not be that clear and incontrovertible.

With this perfunctory prelude to a few features of logicism, the question is whether the Aristotelian corpus affords any textual evidence in support of his allusion and allegiance to the doctrine of logicism. In this regard, one of the most promising sources is Aristotle's epistemological and ontological ruminations and pronouncements in one of his later works, the *Metaphysics*. There is a notable consensus among scholars that Aristotle's *Metaphysics* is intentionally concerned with the problem of skepticism as an integral part of a universal or special science of being. Indeed, his discussion of the Protagorean doctrine, arising out of the problem of conflicting appearances, is purposefully tied to the denial of the *law of non-contradiction*, which in turn is epitomized in the Aristotelian corpus as radical skepticism.

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[13] Russell (1996) p. xv.
[14] *Ibid*.
[15] Tennant (2023).

Prima facie, one may suspect a dissonance here as any discussion of the law of non-contradiction seems to be more ensconced in the domain of logic and its foundation in contrast to a study of the content and details of a universal or special discipline dedicated to the overarching subject of being and existence. However, Aristotle in his pioneering role as the first *metalogician*[16] attempts to shed light on the nature of proof and consequence and, in particular, the status of the law of non-contradiction in his *Metaphysics* with the ultimate aim of demonstrating the *intelligibility* of the broad structure of *reality* in the same breath.[17]

In Aristotle's own articulation, this metaphysical and metalogical interplay and interaction takes place in the following manner:

> Obviously then it is the work of one science to examine being *qua* being, and the attributes which belong to it *qua* being, and the same science will examine not only substances but also their attributes. (*Metaph.* Γ 2.1005a13-16; McKeon 1941, p. 735)

And, lest there is a minimalist or broad understanding of substances and their attributes in this context, Aristotle takes a maximalist or anti-minimalist approach to the universal science of being and adds that:

> We must state whether it belongs to one or different sciences to inquire into the truths which are in *mathematics* called axioms, and into *substance*. Evidently, the inquiry into these also belongs to one science [...] for these truths hold good for everything that is, and not for some special genus apart from others. (*Metaph.* Γ 2-3.1005a18-24; McKeon 1941, pp. 735-6)

Therefore, the question arises, who is qualified to undertake the special science of being in this Aristotelian worldview. To reinforce the point, Aristotle continues by cautioning against two sets of false contenders here. For the first set, he targets mathematicians and, specifically, geometers and arithmeticians:

> since these truths clearly hold good for all things *qua* being (for this is what is common to them), to him who studies being *qua* being belongs the inquiry into these as well. And for this reason no one who is conducting a special inquiry tries to say anything about their truth

[16] Lear (1980).

[17] Similarly, Martin (1964) p. 85: "Aristotelian logic is seen [...] to be a complicated mixture of logic, metalogic and metaphysics, and Aristotelian metaphysics contains logical and metalogical considerations".

or falsity – neither the geometer nor the arithmetician. (*Metaph.* Γ 3.1005a27-31; McKeon 1941, p. 736)

That is, not only are the mathematical axioms not the fundamental principles of what Aristotle's special science is going to ascertain, but they are also not in themselves sufficiently *sui generis* to form an independent set of their own.

For the second set of contenders, Aristotle rebukes natural philosophers for harboring such ontological ambitions. This is quite interesting in view of Aristotle himself being a naturalist *par excellence* as evidenced by his iconoclastic revolt against his master's suprasensible and supernatural entities of the platonic forms. In his dismissal of natural philosophy as the home of being *qua* being, he writes:

Some natural philosophers indeed have done so, and their procedure was intelligible enough; for they thought that they alone were inquiring about the whole of nature and about being. But since there is one kind of thinker who is above even the natural philosopher (for nature is only one particular genus of being), the discussion of these truths also will belong to him whose inquiry is universal. (*Metaph.* Γ 3.1005a31-35; McKeon 1941, p. 736)

In particular, he goes after those who offer the discipline of physics as furnishing the foundational principles of existence. Although Aristotle readily acknowledges the status of physics as "a kind of Wisdom", he chides the advocacy of physics as the special science of being "due to a want of training in *logic* [*analytics*]" (*Metaph.* Γ 3.1005b1-3; McKeon 1941, p. 736).

Thus, the question is which discipline or branch of knowledge has the necessary wherewithal and the *logical* capability to deliver the objectives and goals of the universal or special science of being? Aristotle's answer is unhesitatingly categorical with a tantalizing twist: "Evidently then it belongs to the philosopher, i.e. to him who is studying the nature of all substance, to inquire also into *the principles of syllogism*" (*Metaph.* Γ 3.1005b6; McKeon 1941, p. 736).

The significance of the twist – the reference to the theory of syllogism – can be best appreciated against the backdrop of the forgoing first observation about the project of logicism: the prerequisite or precondition of the availability of a formal deductive system of logic adequate for formalizing the reasoning of one domain into another one.

Against this backdrop, it is important to bear in mind that for Aristotle this appeal to the syllogistic formal system in the context of studying being *qua* being is neither accidental nor incidental. The idea of a reduction process in the discovery, classification, and ordering of the principles of *each* genus of being is a fundamental feature of his formal methodology. Indeed, the burden of his *Prior Analytics* is primarily to provide a formal apparatus through which such determinations and reductions can take place with apodeictic necessity. Aristotle reiterates the same commitment here in the context of the *Metaphysics* again: "he who knows best about each genus must be able to state the most certain principles of his subject, so that he whose subject is existing things *qua* existing must be able to state the most certain principles of all things" (*Metaph.* Γ 3.1005b8-10; McKeon 1941, p. 736).

Before continuing with the logicist reading of the Aristotelian text, an interesting exegetical matter may not be amiss here. In his commentaries on the foregoing *Metaphysics*' passage 1005b1-10, Alexander of Aphrodisias offers two emendations. In the original Aristotelian text, the sequence of argumentation seems to stream thus:

(1) Physics is a kind of wisdom but it is not first philosophy.
(2) People can only engage in the study of truth with a training in or grasp of logic.
(3) It belongs to the philosopher to study the principles of syllogism. And,
(4) the person who knows best about each genus is the one who states its most certain principles.

In his commentary, nevertheless, Alexander of Aphrodisias recommends a juxtaposition of the second and third stages in the series; for, in his logical reconstruction of the Aristotelian reasoning, the second step "follows more closely from" the third one and "would properly be prefixed to" the fourth phase (Alex. *In Metaph.* 267, 19-22; Madigan 1993, p. 47).[18] This, though unwittingly on the part of Alexander, gives more poignancy to the specific

[18] Consequently, some Aristotelian scholars have come to regard this second step of the sequence in this passage as a later addition to the text by Aristotle. See Madigan (1993) p. 154 n. 253.

significance of the syllogistic theory in the manner of a logicist line of thinking with regard to the understanding of Aristotle's ultimate philosophical approach.

The second emendation of Alexander to the text, which again encourages or heightens a logicist approach to Aristotle's outlook here, is his gloss on who is qualified to carry out the demonstrations when drawing on the syllogistic formalization. Alexander writes:

> By principles of syllogism he [Aristotle] means the principles of demonstration, for the axioms are the universal principles of demonstration. For the principles and premises corresponding to each science, which are proper to the things demonstrated in that science, belong to the one who demonstrates in each science; it is their task to know these principles and premises; each of them will do this, while taking from the expert in demonstration, the philosopher, [knowledge of] how one should derive the premises of the demonstration from the properties of that which is being demonstrated, and of how one should combine these premises with one another, as well as of the other matters discussed in the works on demonstration. (Alex. *In Metaph.* 267, 34-268, 6; Madigan 1993, p. 47)

In other words, "the one who demonstrates," in Alexander's commentary, is not merely any practitioner who demonstrates in this or that science, but a *bona fide* expert in demonstration as specified in the Aristotelian analytical text. Consequently, this appears to imply that the *first philosopher* is strictly speaking the *logician*. Moreover, Alexander's circumscription of first philosophy to logic is not only going to be congenial to the proponents of logicism but also proffers a wider perspective in terms of situating Aristotle's approach in a new light.

Now, picking up the earlier thread in Aristotle's own text in terms of its logicist leanings, we may pose the question: what is after all the outcome of the study of being as being by inquiring into "the principles of syllogism"? The result is a principle, remarks Aristotle, that "is the most certain of all": "*It is, that the same attribute cannot at the same time belong and not belong to the same subject and in the same respect*": that is, the law of non-contradiction. (*Metaph.* Γ 3.1005b17-20; McKeon 1941, p. 736). Yet, to leave no room for doubt as to the core fundamentality and centrality of this principle *vis-à-vis* any other principles, including *mathematical* ones, Aristotle sharpens his 'logicist' stance by the following observation:

> This, then, is the most certain of all principles [...] *that all who are carrying out a demonstration reduce it to this as an ultimate belief; for this is naturally the starting-point even for all the other axioms*. (*Metaph.* Γ 3.1005b22, 1005b31-34; McKeon 1941, pp. 736-7, emphasis added)

From a comparative point of view, it is worth noting Leibniz's take on the law of non-contradiction here. He writes:

> The great foundation of mathematics is the *principle of contradiction* [...] This single principle is sufficient to demonstrate every part of arithmetic and geometry, that is, all mathematical principles.[19]

This statement of Leibniz not only displays an exact echo of Aristotle's approach to the law of non-contradiction as presented in the preceding passage from the *Metaphysics* but also highlights the logicist implication of it in an important and immediate manner.

Given such a reading of the Aristotelian text, the philosophical upshot is that, in Aristotle's ontology, what ultimately underwrites being and existence is logic, or, more specifically, the law of non-contradiction. This thus paves the way for the claim that metaphysics and metalogic seem to be intrinsically coextensive in the Aristotelian architecture. On this basis, it may not therefore be an anachronism to think of Aristotle as an early proponent or a precursor of logicism, except on a grander scale than its circumscribed mathematical variety as presented in the works of Leibniz, Frege, Russell and later neo-logicists when it comes to the overall ontological structure of reality.

Philosophically, however, there is a question or puzzle here that deserves some attention, albeit very briefly, which forms the concluding part of this paper. If, as I contend, there is a logicist undertone, if not an outright overtone, in Aristotle's *Metaphysics* in arguing for the reduction of *all axioms* to the most certain of all principles, the law of non-contradiction, why do not we see an application or extension of this project either in a wholesale or piecemeal fashion by Aristotle himself or his disciples and successors? Other than an exegetical clarification by Alexander of Aphrodisias that the first philosopher in Aristotle's *Metaphysics* is intended to be the logician, the

[19] Loemker (1969) p. 677.

prototypical Aristotelian logicist project appears to be immediately stalled after its inauguration.

There may be two explanations for the arrested development of the Aristotelian logicism. One thought is the apparent overcommitment to certain *rigid* and *irreducible* categories within the Aristotelian conceptual architecture such that the idea of a reduction process in the discovery, classification, and ordering of the principles of *each* genus of being becomes an inflexible fundamental feature of Aristotle's formal methodology. Indeed, the burden of his *Prior Analytics* is primarily to provide a formal apparatus through which such determinations and reductions can be established with necessity. In the *Posterior Analytics*, in particular, he seems to set up such stringent conditions for conceptual reducibility that a logicist reduction becomes for all intents and purposes a punitive practice. Specifically, on the relationship between geometry and arithmetic, not only does Aristotle express full recognition and awareness of the purported possibility of reducing geometrical truths to arithmetical ones, but he also expressly argues against it:

> we cannot in demonstrating pass from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic. For there are three elements in demonstration: (1) what is proved, the conclusion – an attribute inhering essentially in a genus; (2) the axioms, i.e. axioms which are premisses of demonstration; (3) the subject-genus whose attributes, i.e. essential properties, are revealed by the demonstration. The axioms which are premisses of demonstration may be identical in two or more sciences: but in the case of two different genera such as arithmetic and geometry you cannot apply arithmetical demonstration to the properties of magnitudes unless the magnitudes in question are numbers. (*An. Post.* 74a37-75b5; McKeon 1941, pp. 121-2)

Although in his general discussion of the topic of conceptual reduction Aristotle allows the distinction between *superior* and *subordinate* in relation to two domains or sciences such that "optical problems are subordinated to geometry, mechanical problems to stereometry, harmonic problems to arithmetic, the data of observation to astronomy" (*An. Post.* 78b39-79a1; McKeon 1941, p. 130), the inflexibility and inelasticity of his categorical classifications appear to pose an obstacle to satisfying the second characteristic of the logicist project mentioned earlier. That is, the requirement of providing a transparent process of *conceptual reduction* whereby the concepts of the prospective target domain for reduction can be defined in terms of the concepts of, what ironically Aristotle himself calls, a "more exact than and prior to" domain (*An. Post.* 87a33; McKeon 1941, p. 153).

The other explanation for the underdevelopment of logicism in Aristotle and his heirs may be sought in the overgeneralization of the syllogistic theory to all domains of knowledge in the sense of neglecting to recognize the prevalence of *asyllogistic* deductive reasoning and argumentation. This could be partly attributed to overestimating both (i) the applicability of Aristotle's dichotomy between *perfect* and *imperfect* syllogism and thereby ambitiously assimilating all asyllogistic reasoning to species of imperfect syllogism and (ii) the ability to convert the presumed imperfect cases of syllogism to the perfect one. Thus, Russell's remark is fitting here that the "syllogism in all its figures belongs to Symbolic Logic, and would be the whole subject if all deduction were syllogistic, as the scholastic tradition supposed. It is from the recognition of asyllogistic inferences that modern Symbolic Logic, from Leibniz onward, has derived the motive to progress".[20]

Bibliography

Black, M. (1958) *The Nature of Mathematics* (1933). London: Routledge & Kegan Paul.

Demopoulos, W. and Clark, P. (2007) 'The Logicism of Frege, Dedekind, and Russell' in Shapiro, S. (ed.) *The Oxford Handbook of the Philosophy of Mathematics and Logic*. Oxford: Oxford University Press, pp. 166-202.

Febvre, L. (1982) *The Problem of Unbelief in the Sixteenth Century: The Religion of Rabelais*. B. Gottlieb (trans.). Cambridge (MA): Harvard University Press.

Franchella, M. (2019) 'Shaping the Enemy: Foundational Labelling by L.E.J. Brouwer and A. Heyting', *History and Philosophy of Logic*, 40 (2), pp. 152-181.

Frege, G. (1978) *The Foundations of Arithmetic* (1884). J. L. Austin (trans.). Oxford: Blackwell.

Frege, G. (1979) *Posthumous Writings*. Oxford: Blackwell.

[20] Russell (1996) p. 10.

Frege, G. (2007) *The Foundations of Arithmetic* (1884). D. Jacquette (trans.). New York: Longman.

Lear, J. (1980) *Aristotle and Logical Theory*. Cambridge: Cambridge University Press.

Leibniz, G.W. (1985) *New Essays on Human Understanding*. P. Remnant and J. Bennett (trans. & ed.). Cambridge: Cambridge University Press.

Loemker, L.E. (ed.) (1969) *Leibniz: Philosophical Papers and Letters*. Second Edition. Dordrecht: D. Reidel Publishing.

Madigan, A. (1993) *Alexander of Aphrodisias. On Aristotle Metaphysics 4*. London: Duckworth.

Martin, G. (1964) *Leibniz: Logic and Metaphysics*. Manchester: Manchester University Press.

McKeon, R. (ed.) (1941) *The Basic Works of Aristotle*. New York: Random House.

Reck, E. (2013) 'Frege, Dedekind, and the Origins of Logicism', *History and Philosophy of Logic*, 34 (3), pp. 242-265.

Russell, B. (1919) *Introduction to Mathematical Philosophy*. London: Allen and Unwin.

Russell, B. (1958) *A Critical Exposition of the Philosophy of Leibniz*. London: Allen and Unwin.

Russell, B. (1996) *The Principle of Mathematics* (1903). New York & London: W. W. Norton & Company.

Stein, H. (1998) 'Logicism' in Craig, E. (ed.) *Routledge Encyclopedia of Philosophy*. London: Routledge, Vol. 5, pp. 811-817.

Tennant, N. (2023) 'Logicism and Neologicism', in E.N. Zalta and U. Nodelman (eds.), *The Stanford Encyclopedia of Philosophy*. Available at https://plato.stanford.edu/archives/win2023/entries/logicism/